DESIGN OF Z-PURLINS: PART 2
Design methods given in Eurocode EN 1993-1-3

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INTRODUCTION

The scope of part 1-3 of EN 1993 also known as Eurocode 3 is the design of cold-formed members and sheetings. Depending on the complexity of roofs different methods to design Z-purlins are given in this standard.
The paper at hand contains an overview of possible methods, the workflow and the effects of the advanced method in detail. Required are the cross section properties for the gross cross section and the effective cross sections as described in part 1.

1 FIELD OF APPLICATION: METHODS

The general design concept is given by chapter 6. Special rules for purlins are subject of chapter 10 and Annex E. Purlins can be designed by:

- Simplified Method (annex E)
- Basic Method (10.1.3 ff)
- Advanced Method using 2nd order analysis (10.1.2 (1) +(6) and 10.1.3)
- Design by testing according to Annex A.

Table 1. Design methods given in EN 1993-1-3 and their limits

chapter	remarks	moments	vertical force	normal force	spans	action	anti sagging bars	sleeve/over-lapping joint	thickness of purlins (EI) [2]
Annex E	using factors		no	–	constant	uniform loading	–	–	constant
10.1.3			yes	no	constant	(variable)	up to 4	yes [1]	constant
10.1.2 +	2nd order analysis	about both axes	yes	yes	variable	(variable)	yes	yes [1]	variable
Annex A	testing – not treated here								

[1] test of (flexural) stiffness of sleeve or overlap necessary
[2] end spans thicker purlins

Due to the limits for the simplified and the basic method, the necessity of the advanced method is obvious. For longer roofs we need to overlap or sleeve purlins, for longer spans the requirement of anti-sagging bars is evident. If the end spans are shorter or the thickness of the end span purlins are chosen thicker only the advanced method based upon 2nd order analysis is possible.

The advanced method only can handle unequal uniformly distributed loads for each span. Even this method has its limit: For example concentrated loads resulting from roof trimmers are not implicated due to the determination procedure of k_h.

2 STRUCTURAL SYSTEM

In difference to the standard check of sections about major and minor axis, the check for Z-purlins will be done about the axis parallel to the roof and perpendicular using an additional loading acting about the major axis of the free flange.

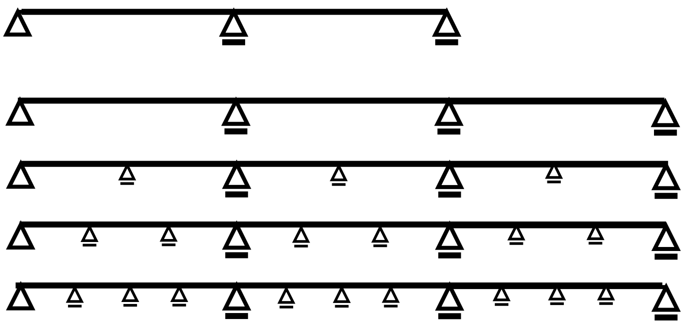


Fig. 1. Supports for free flange by purlins and anti-sagging bars. examples: two-span purlin, three span purlin without / with one / with two /with three anti-sagging bars

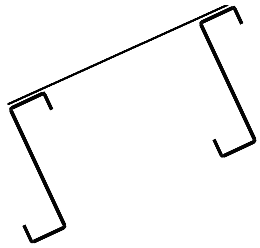


Fig. 2. Orientation of Z-purlins

3 CHECK OF PURLIN

Normal stress resulting from normal force and bending about two axis can be determined about any pair of perpendicular axes in general as follows:

$$\sigma_x(x) = \frac{N}{A} + \frac{\left(M_y(x)\cdot I_z - M_z(x)\cdot I_{yz}\right)\cdot z - \left(M_z(x)\cdot I_y - M_y(x)\cdot I_{yz}\right)\cdot y}{I_y\cdot I_z - I_{yz}^{\;2}}. \tag{1}$$

If the moments are calculated about the major (1-1) and the minor (2-2) axes the stress can be determined in the well known format:

$$\sigma_x(x) = \frac{N}{A} + \frac{M_y(x)\cdot z}{I_y} - \frac{M_z(x)\cdot y}{I_z} \tag{2}$$

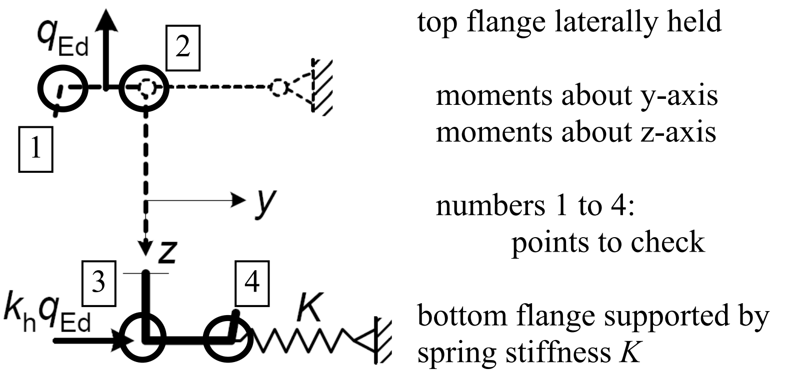


Fig. 3. Model for actions, check method and spring stiffness

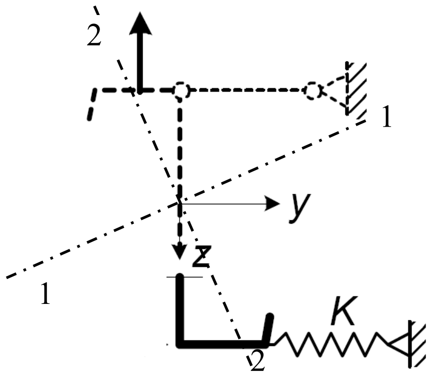


Fig. 4. Major and minor axis

The method chosen to do the check according to EC 3-1-3 for Z-purlins is to determine the stress using the bending about the sheeting parallel y-axis plus an additional load acting in y-direction in the axis of the free flange.

For this check the first step is to get this additional load for the free flange of the Z-purlins acting on the roof-parallel y-axis using formula (10.4) of EC3-1-3:

$$q_{h,Ed} = k_h \cdot q_{Ed} \tag{3}$$

where q uniformly distributed load [kN/m]

k_h factor

Because k_h is depending on the direction of the load (uplift or downwards) the sum of all loads has to be considered:

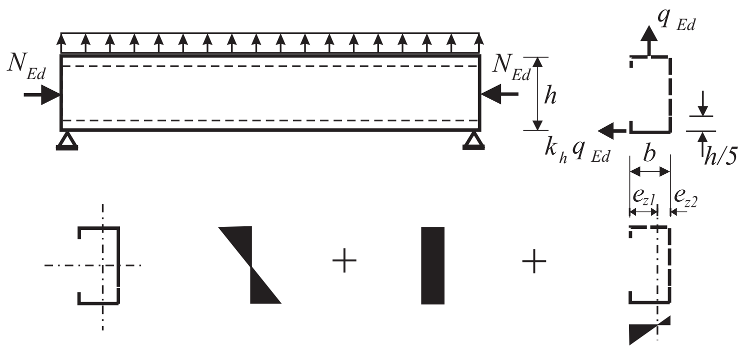
$$q_{j,h,Ed} = k_h \cdot \sum^{i} q_{i,Ed} = k_h \cdot \sum^{i} \gamma_i \cdot \psi_i \cdot q_{i,Ek} \tag{4}$$

where ψ combination factor

γ partial safety factor

j load combination j with i load cases

This means that load cases cannot be super-positioned for the action about the y-axis. This is analogue to the approach for second order analysis. Due to the achieved moments about the y-axis the free flange gets a variable normal force over the span.



$$\sigma_{\max,Ed} = \frac{M_{y,Ed}}{W_{eff,y}} + \frac{N_{Ed}}{A_{eff}} + \frac{M_{fz,Ed}}{W_{fz}} \le \frac{f_y}{\gamma_M}$$

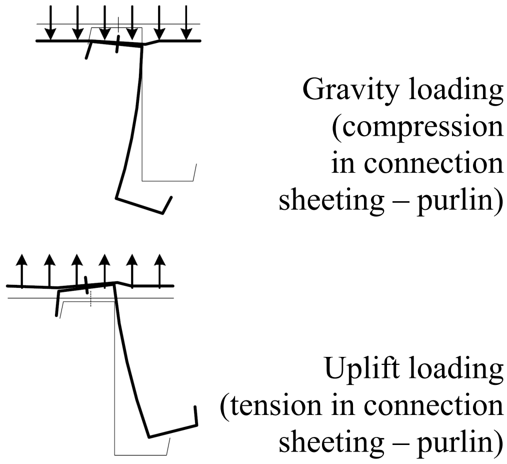
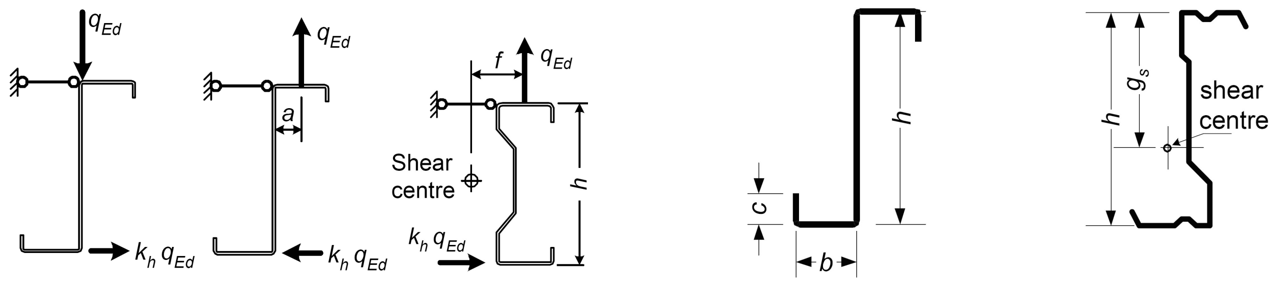


Fig. 5. Super-position of primary and secondary action

Fig. 6. Loading direction

The value of k_h depends on the shape of the cross section and the direction of the action:



$$k_h = k_{h0} \qquad k_h = k_{h0} - a/h \qquad k_h = k_{h0} - f/h \qquad k_{h0} = \frac{h \cdot t \cdot \left(b^2 + 2 \cdot b \cdot c - 2 \cdot b \cdot c^2/h\right)}{4 \cdot I_y}; \quad k_{h0} = \frac{I_{yz}}{I_y} \cdot \frac{g_s}{h}$$

Fig. 7. Determination of k_h: load direction and point of action

For the basic and the advanced method the following checks have to be done:

M_y, N: (10.3a)

$$\sigma_{\max,Ed} = \frac{M_{y,Ed}}{W_{eff,y}} + \frac{N_{Ed}}{A_{eff}} \le \frac{f_y}{\gamma_M}, \tag{5}$$

free flange M_y, M_z, N: (10.3b)

$$\sigma_{\max,Ed} = \frac{M_{y,Ed}}{W_{eff,y}} + \frac{N_{Ed}}{A_{eff}} + \frac{M_{fz,Ed}}{W_{fz}} \le \frac{f_y}{\gamma_M}, \tag{6}$$

combined *M, N, V:* (6.27) $$\frac{N_{Ed}}{N_{Rd}}+\frac{M_{y,Ed}}{M_{y,Rd}}+\left(1-\frac{M_{f,Rd}}{M_{pl,Rd}}\right)\cdot\left(\frac{2\cdot V_{Ed}}{V_{w,Rd}}-1\right)^2 \leq 1. \qquad (7)$$

For the basic method lateral torsional buckling (10.7) has to be fulfilled:

$$\frac{1}{\chi_{LT}}\cdot\left(\frac{M_{y,Ed}}{W_{eff,y}}+\frac{N_{Ed}}{A_{eff}}\right)+\frac{M_{fz,Rd}}{W_{fz}}\leq\frac{f_{yb}}{f_{M1}}. \qquad (8)$$

Although the free flange gets additional action by $k_h\ q_{Ed}$ the supported upper flange is held by the trapezoidal sheet and therefore gets no additional stress. For these checks always the according effective cross-section values have to be taken except for the section modulus of the free flange W_{fz}.

4 SPRING STIFFNESS

Requirement for all design methods is to determine the spring stiffness by which the free flange is supported. Determination of lateral spring stiffness is a product of the rotational stiffness coming from the local bending of the web, the stiffness of roof sheeting and the stiffness of fasteners sheeting/purlin.

4.1 Rotational spring stiffness of Z-purlins

Therefore the rotational stiffness has to be determined first:

$$c_D=\frac{1}{\left(1/c_{D,A}+1/c_{D,C}\right)} \qquad (9)$$

where c_D rotational spring stiffness per unit length [kNm/m]
$c_{D,A}$ rotational spring stiffness of connection sheeting / purlin
$c_{D,C}$ rotational spring stiffness corresponding to the flexural stiffness of the sheeting

$$c_{D,A}=c_{100}\cdot k_{ba}\cdot k_t\cdot k_{bR}\cdot k_A\cdot k_{bT} \qquad (10)$$

where c_{100} rotational coefficient for trapezoidal sheeting (table 10.3 of EC3-1-3)
k_{ba} width factor of purlin flange
k_t thickness factor (constant values, linear between given limits)
k_{bR} corrugation width factor of sheeting
k_A supporting force factor (from sheeting to purlin)
k_{bT} connection factor

$$c_{D,C}=\frac{k\cdot E\cdot I_{eff}}{s} \qquad (11)$$

where k structural system coefficient of sheeting
E modulus of elasticity (Young's modulus)
I_{eff} effective moment of inertia per unit width of the sheeting
s span of the sheeting

4.2 Spring stiffness per unit length of the free flange

Having gotten the rotational spring stiffness, the lateral spring stiffness for the support of the free flange can be evaluated as follows:

$$1/K=\frac{4\cdot(1-\nu^2)\cdot h^2\cdot(h_d+b_{mod})}{E\cdot t^3}+\frac{h^2}{c_D} \qquad (12)$$

where K lateral spring stiffness per unit length
ν restraint coefficient
h overall height of the purlin

a	distance from sheet to purlin fastener to the purlin web
b	width of the purlin flange connected to the sheeting
b_{mod}	distance depending on uplift or downwards actions
h_d	developed height of the purlin web

5 FREE FLANGE

With the section properties and the spring stiffness all necessary internal forces can be calculated:

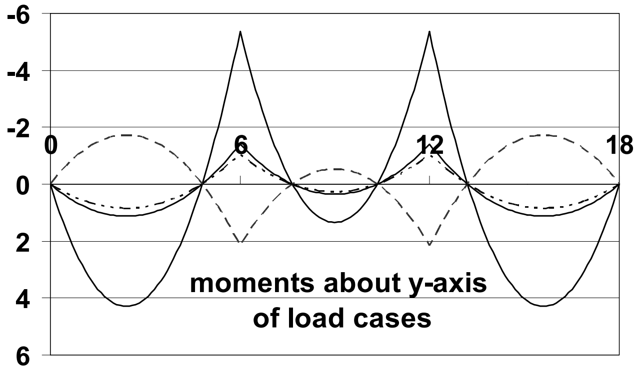


Fig. 8. Moments of load cases about y-axis

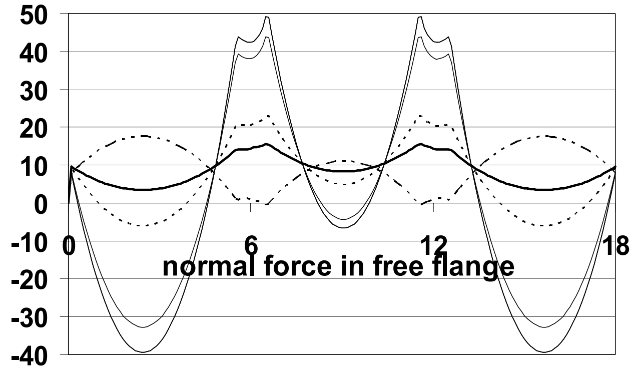


Fig. 9. Normal force in free flange resulting from normal force and bending about y-axis

5.1 Imperfections

For impair spans we get symmetrical equivalent imperfections. For pair spans or pair spans between ant-sagging bars the result is an unsymmetrical equivalent imperfection. This results in unsymmetrical moments.

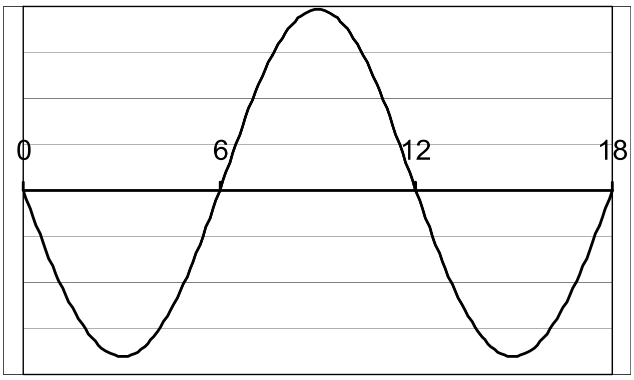


Fig. 10. Imperfection for free flange of a three span continuous girder purlin

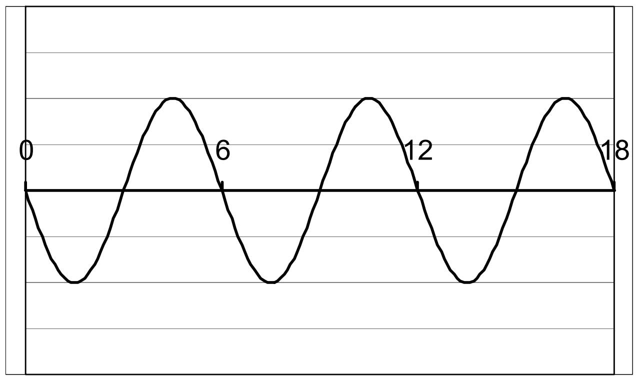


Fig. 11. Three span continuous girder purlin with one anti-sagging bar per span

5.2 First order analysis

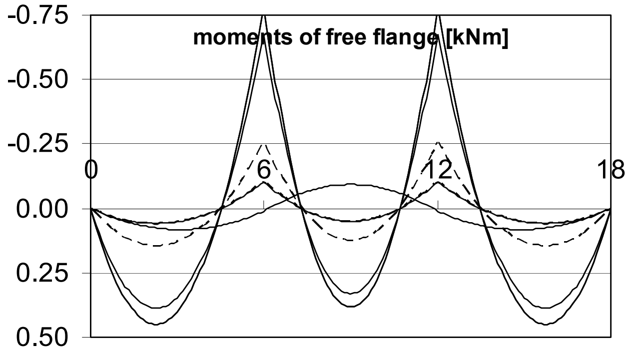


Fig. 12. Moments: no anti-sagging bars

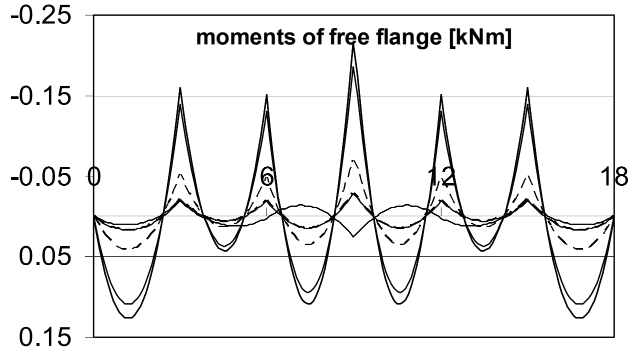


Fig. 13. Moments: one anti-sagging bar per span

5.3 Second order analysis

Second order analysis with variable compression (or tension) parts as well as unsymmetrical imperfections lead to peculiar moment graphs (see fig. *14* and *15*).

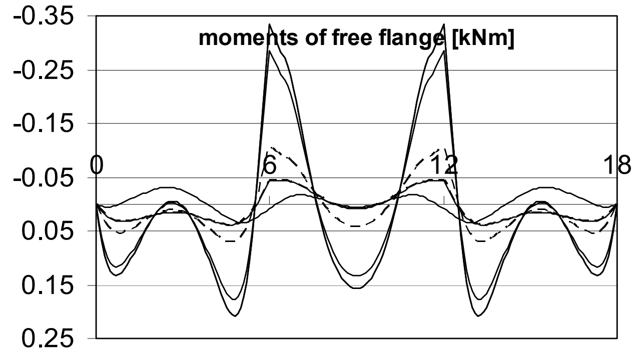


Fig. 14. Moments of free flange about z-axis: no anti-sagging bars, second order analysis

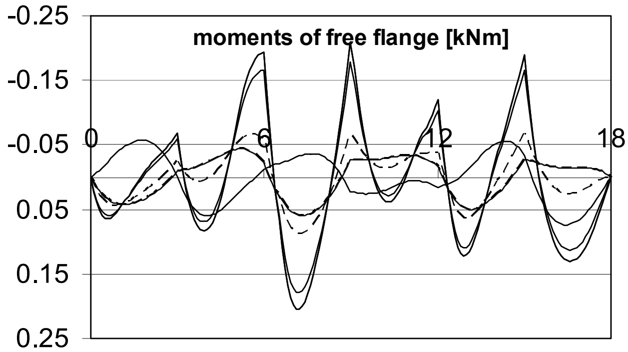


Fig. 15. Moments of free flange about z-axis: one anti-sagging bar per span, second order analysis

6 ADDITIONAL REMARKS

6.1 Strengthening of end spans by thickness increase

Continuous girder system have the disadvantage of greater sagging moments in the end spans. Resulting demand is to strengthen the end span by shorten them or to use thicker purlins. Shorter spans means that the modular grid is discontinued which is not possible in many cases. For proper developed purlins it is possible to combine purlins with small thickness differences.

6.2 Sleeves and Overlaps

One and two span girders are normally done by one purlin only. Continuous girder purlins will be joined by overlapping or sleeves. Overlapping purlins are done by alternating positive and negative positions of unsymmetrical Z-shapes.

REFERENCES

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